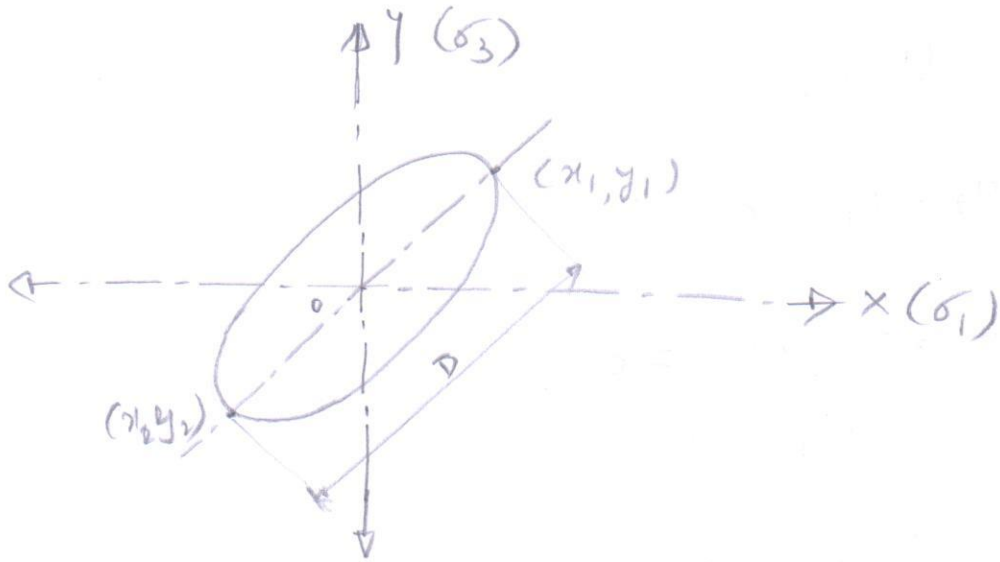
Chapter: 2

Not really qualifies

* Yield Locus (2D) : Prove that: $\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3 = \sigma_{ys}^2$ is an eq^n of ellispse with ellipse major axis $2\sqrt{2}\,\sigma_{ys}$ & Minor axis $2\sqrt{2/3}\,\sigma_{ys}$ ~~at~~ 45° with the horizontal axis.

⇒



Let $x = \sigma_1$

$y = \sigma_3$ & σ_{ys} = Const = c

$$\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3 = \sigma_{ys}^2$$

$$\therefore x^2 + y^2 - xy = c^2 \quad \text{——— (1)}$$

If (x_1, y_1) & (x_2, y_2) → Two points on ~~ellipse~~ curve (lies on its major & (or) minor axis) then,

(as it is not known whether ellipse eq^n or circle eq^n)

$$x_1^2 + y_1^2 - x_1 y_1 = c^2$$

$$x_2^2 + y_2^2 - x_2 y_2 = c^2$$

& $y_1 = m x_1$ & $y_2 = m x_2$

$$\therefore \frac{y_1}{x_1} = \frac{y_2}{x_2} \quad \text{——— (2)}$$

$$\therefore D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$\therefore D^2 = (x_2-x_1)^2 + (y_2-y_1)^2 \quad \text{———} \quad (3)$$

from (1) & (2),

$$x_1^2 + y_2^2 - xy = c$$

$$\therefore x^2 + m^2x^2 - x\,mx = c$$

$$\therefore x^2(1+m^2-m) = c$$

$$\therefore x = \pm\sqrt{\frac{c}{(m^2-m+1)}}$$

$$\left.\begin{aligned} \therefore x_1 &= +\sqrt{\frac{c}{m^2-m+1}} \\ \therefore x_2 &= -\sqrt{\frac{c}{m^2-m+1}} \end{aligned}\right\} \text{Using in (3),}$$

$$\Rightarrow D^2 = (x_2-x_1)^2 + (mx_2 - mx_1)^2$$

$$= (x_2-x_1)^2 + m^2(x_2-x_1)^2$$

$$= (x_2-x_1)^2(1+m^2)$$

$$= \cancel{(x_2-x_1)^2}\left[-\left\{\sqrt{\frac{c}{m^2-m+1}} + \sqrt{\frac{c}{m^2-m+1}}\right\}\right]^2(1+m^2)$$

$$D^2 = \left[\frac{4c}{\ldots}\right]\{1+m^2\}$$

For maximum & minimum value of $D \rightarrow$ $\underline{m = ?}$

$$\therefore \frac{d}{dm}(D^2) = 2c\left[\frac{(m^2-m+1)(2m)-(m^2+1)(2m-1)}{(m^2-m+1)^2}\right]$$

$$0 = 2c\left[\frac{2m^3-2m^2+2m-2m^3+m^2-2m+1}{(m^2-m+1)^2}\right]$$

$$0 = 2c\left[\frac{-m^2+1}{(m^2-m+1)^2}\right]$$

$$\therefore 2c(1-m^2) = 0$$

$$c \neq 0 \rightarrow \sigma_{ys}^2$$

$$\therefore (1-m^2) = 0$$

$$\therefore m^2 = 1$$

$$\therefore \boxed{m = \pm 1} \quad \text{——— (5)}$$

* $\underline{m=1}$ $\therefore$ $D = \sqrt{\frac{4c}{(m^2-m+1)}(1+m^2)}$

$$= 2\sqrt{c}\sqrt{\frac{1+1}{1}}$$

$\underline{c = \sigma_{ys}^2}$

* $D_{max} = 2\sqrt{2}\,\sigma_{ys}$

* $\underline{m=-1}$ $\Rightarrow D = 2\sqrt{c}\sqrt{\frac{2}{}}$

$\therefore$ D_{max} = major axis of ellipse = $2\sqrt{2}\,\sigma_{ys}$

D_{min} = Minor axis of ellipse = $2\sqrt{\frac{2}{3}}\,\sigma_{ys}$

from ⑤, $m = \pm 1 \Rightarrow$ Slope $\Rightarrow \tan\theta$

$\therefore \tan\theta = \pm 1 \Rightarrow$ ~~D~~ makes $\Rightarrow \theta = 45^\circ, 225^\circ$

Line joining (x_1, y_1) & $(x_2\, y_2) \Rightarrow D \Rightarrow$ axis of ellipse makes an angle of 45° with horizontal axis.

0.5